Weekend Activity: To Converge or Not to Converge...

Complete this activity by Nov 7. A **convergence test** is a method for determining whether a series converges or diverges. Each method can be useful on different types of series. Some of the tests are not always practical, or can give inconclusive results. They generally reduce the problem to another that you may be able to solve: the convergence of a limit, another simpler series, or an improper integral.

A) Attach this to last weekend's activity. Also attach some extra paper for C-E.
B) Use your experience from the last activity to fill in the blanks below. You will produce your own versions of four convergence tests.
C) For each test, come up with a series not already in this packet that the test proves is convergent or divergent
D) Come up with a series or situation for which the test is not useful/inconclusive.
E) Match each test to the best name. Given in no particular order, the names of these four tests are: the integral test, the absolute convergence test, the comparison test, and the nth term test.

Note: When it comes to convergence, only the long term behavior of the series matters. To make the tests a little more general, we restrict our attention to the terms beyond a certain integer N.

1) Make the most general test that you can, based on your answer to 10b:

If $\lim_{n\to\infty} a_n$ ______________, then $\sum_{n=1}^{\infty} a_n$ ______________.

2) Create a test based on the technique you used in question 11: Let a_n and b_n be two sequences and N a natural number be such that ______________ for all $n \geq N$. Then if ______________ converges, then ______________ converges; if ______________ diverges, then ______________ diverges.

3) Create a test based on the technique you used in question 12: Let $f(x)$ be a continuous, positive, decreasing function for all $x \geq N$. Let a_n be a sequence such that ______________ for all $n \geq N$. Then if ______________ converges, ______________ converges. Additionally, if ______________ diverges, then ______________ diverges. (you can use overestimating rectangles to prove this second part)

4) Create a test based on your solution to question 13: Let $\sum_{i=1}^{\infty} a_n$ be an alternating series. Then if ______________ converges, $\sum_{i=1}^{\infty} a_n$ converges.